Mathematical description of a bounded oil reservoir with a horizontal well: late time flow period

Nzomo T. K.^{1*}, Adewole S. E.², Awuor K. O.³, Oyoo D. O.⁴

^{1,3}Department of Mathematics, Kenyatta University, Nairobi, Kenya

²Department of Petroleum Engineering, University of Benin, Benin City, Nigeria

⁴Department of Gas and Petroleum Engineering, Kenyatta University, Nairobi, Kenya

^{1*}Corresponding Author. Email: nzomotimothy@gmail.com

Received: 24 June 2020 / Accepted: 10 November 2020 / Published online: 29th January 2021

ABSTRACT

When horizontal wells are compared with verticals wells, their production is always higher. If their performance can be improved, they can even be more productive. Considering a horizontal well in a completely bounded oil reservoir, when the well has been producing for some time and the effect of the boundaries is evident on the flow, the pressure distribution can be approximated by considering the effects of the boundaries on the flow. Considering when a pseudosteady state flow is attained this study presents a mathematical model for approximating pressure distribution for late time for a horizontal well in an oil reservoir with sealed boundaries. We use appropriate Source and Green's functions to develop the model. The model developed show that when the flow reaches all the boundaries a pseudosteady state flow is attained and thus pressure distribution is influenced by the oil reservoir geometry especially its width and length. Considering that the thickness of the oil reservoir will be small compared to the length of the well, the oil reservoir width and length will determine the pressure response. This will influence the flow period occurring. By considering all aspects of the flow, the model can be applied to approximate the pressure distribution for as long as the well can continue producing.

Key Words: Horizontal well, pressure distribution, bounded oil reservoir, permeability, long time approximation.

INTRODUCTION

For horizontal wells, the many aspects of the flow can affect the transient flow behavior. The oil reservoir boundaries based on the well location and the anisotropic permeabilities are parameters that have to be considered when developing a model for pressure distribution. To enhance productivity it's important to develop detailed mathematical models that can be used to approximate pressure distribution at any given time of production. The objective of carrying out this study thus is to develop an in depth mathematical model that considers a horizontal well in a completely bounded oil reservoir that can be used to approximate pressure distribution when a well has produced for a sometime and the effects of the oil reservoir boundaries are evident.

Different literature provides models for horizontal wells. Carslaw and Jaeger (1959) came up with results that could be applied in transient flow which has continued to influence fluid flow modelling. Advances in well test analysis have continued to be based on the monograph developed by Mathews and Russell (1967). (Gringarten & Ramey, 1973) detailed the use of source and Green's functions in solving unsteady flows. Lee (1982) discussed the use of source and Green's functions to model pressure in a bounded oil reservoir during early time. Over the years different authors used these source functions to model pressure (Daviau et al., 1985; Clonts & Ramey, 1986; Ozkan et al., 1987; Odeh & Babu, 1989; Carvalho & Rosa, 1988). Adewole (2009, 2010) used source

and Green's functions to develop solutions for pressure distribution for bounded oil reservoirs. Al Rbeawi and Tiab (2013) considered a multi-boundary system and applied the source functions to analyze transient behavior in horizontal wells. Most of these studies have developed mathematical models that consider isotropic permeability which is not always the case for practical horizontal wells and where anisotropy is considered, many assumptions limit the effectiveness of the model. We intend to derive a mathematical model that accounts for anisotropic permeability for a horizontal well considering a bounded oil reservoir for late time flow period.

Mathematical Description

To develop the required mathematical model we start by considering the diffusivity equation (Lee, 1982),

$$\nabla^2 P = \frac{\phi \mu c}{k} \frac{\partial P}{\partial t} \quad (1)$$

Where P is pressure, μ is fluid viscosity, c is compressibility, ϕ is porosity k is the formation permeability and t is time.

Keeping porosity constant, ignoring gravitational forces and considering a slightly compressible fluid, when we consider permeability anisotropy, the heterogeneous three dimensional diffusivity equation becomes (Odeh & Babu, 1989),

$$k_x \frac{\partial^2 P}{\partial x^2} + k_y \frac{\partial^2 P}{\partial y^2} + k_z \frac{\partial^2 P}{\partial z^2} = \phi \mu c_t \frac{\partial P}{\partial t} \quad (2)$$

Where k_x , k_y and k_z are the respective permeabilities along the x-axis, y-axis and z-axis respectively and c_t is the total formation compressibility.

Dimensionless Pressure

The Dimensionless pressure is given by Mathews and Russell (1967)

$$P_D = \frac{k h \Delta P}{141.2 q \mu B} \quad (3)$$

Which can be written as

$$P_D = \frac{2 \pi k h}{q \mu} \Delta P \quad (4)$$

Or

$$P_D = 2 \pi h_D \int_0^{t_D} s(x_D, \tau_D) \cdot s(y_D, \tau_D) \cdot s(z_D, \tau_D) d\tau_D \quad (5)$$

With;

$$\Delta P = \frac{q}{\phi c_t L} \int_0^t s(x, y, z, t) dt \quad (6)$$

Where q is the flow rate, B is the formation volume factor and τ is a dummy variable for time, t .

The product $s(x, y, z, t)$ is obtained using the Newman product rule (Gringarten & Ramey, 1973) and is given by

$$s(x, y, z, t) = s(x, t) \cdot s(y, t) \cdot s(z, t) \quad (7)$$

Where $s(x, t)$, $s(y, t)$ and $s(z, t)$ are appropriate source and Green's functions.

METHODOLOGY

Description of the Physical Model

The physical model used to develop this model is as shown in Figure 1. The drainage volume considered assumes an oil reservoir whose length is x_e taken in the x-axis, the reservoir width is considered to be y_e taken along y-axis and a reservoir thickness of h along the z-axis. We consider a horizontal well having a length L parallel to the x-axis. The well is centrally located at (x_w, y_w, z_w) stretching a length of $L/2$ from x_w in both directions along x.

Source Functions

As shown in the physical model in Figure 1, the horizontal well indicated can be considered to be a horizontal line source of length L at (x_w, y_w, z_w) and thus the source function is a result of intersection of three sources, (Gringarten & Ramey, 1973) namely;

- A source function located at $y = y_w$ in a reservoir having a width y_e as shown in Figure 2.
- A source function located at $x = x_w$ in a reservoir having a length x_e as shown in Figure 3.
- A source function located at $z = z_w$ in a reservoir having a thickness h as shown in Figure 4.

Figure 1: Physical Model of a Horizontal Well in a Rectangular Drainage Volume

Figure 2: Infinite-plane Source along y-axis

Figure 3: Infinite-slab Source along x-axis

Figure 4: Infinite-plane source along the z-axis

RESULTS AND DISCUSSION

When all the boundaries have started affecting the flow then we consider a pseudosteady state. In this flow period, we consider the following source functions as given by Gringarten & Ramey (1973).

(a) The instantaneous source function along x-axis given as;

$$s(x, t) = \frac{L}{x_e} \left\{ 1 + \frac{4x_e}{\pi L} \sum_{n=1}^{\infty} \frac{1}{n} \exp \left[-\frac{n^2 \pi^2 \eta_x t}{x_e^2} \right] \sin \frac{n\pi L}{2x_e} \cos \frac{n\pi x_w}{x_e} \cos \frac{n\pi x}{x_e} \right\} \quad (8)$$

(b) The instantaneous source function along y-axis defined given as;

$$s(y, t) = \frac{1}{y_e} \left\{ 1 + 2 \sum_{m=1}^{\infty} \exp \left[-\frac{m^2 \pi^2 \eta_y t}{y_e^2} \right] \cos \frac{m\pi y_w}{y_e} \cos \frac{m\pi y}{y_e} \right\} \quad (9)$$

(c) And the instantaneous source function along z-axis given as;

$$s(z, t) = \frac{1}{h} \left\{ 1 + 2 \sum_{l=1}^{\infty} \exp \left[-\frac{l^2 \pi^2 \eta_z t}{h^2} \right] \cos \frac{l\pi z_w}{h} \cos \frac{l\pi z}{h} \right\} \quad (10)$$

Substituting in equation (7) we obtain;

$$S(x, y, z, t) = \frac{L}{x_e} \left\{ 1 + \frac{4x_e}{\pi L} \sum_{n=1}^{\infty} \frac{1}{n} \exp \left[-\frac{n^2 \pi^2 \eta_x t}{x_e^2} \right] \sin \frac{n\pi L}{2x_e} \cos \frac{n\pi x_w}{x_e} \cos \frac{n\pi x}{x_e} \right\} \cdot \frac{1}{y_e} \left\{ 1 + 2 \sum_{m=1}^{\infty} \exp \left[-\frac{m^2 \pi^2 \eta_y t}{y_e^2} \right] \cos \frac{m\pi y_w}{y_e} \cos \frac{m\pi y}{y_e} \right\} \cdot \frac{1}{h} \left\{ 1 + 2 \sum_{l=1}^{\infty} \exp \left[-\frac{l^2 \pi^2 \eta_z t}{h^2} \right] \cos \frac{l\pi z_w}{h} \cos \frac{l\pi z}{h} \right\} \quad (11)$$

Or

$$S(x, y, z, t) = \frac{L}{x_e y_e h} \left\{ 1 + \frac{4x_e}{\pi L} \sum_{n=1}^{\infty} \frac{1}{n} \exp \left[-\frac{n^2 \pi^2 \eta_x t}{x_e^2} \right] \sin \frac{n\pi L}{2x_e} \cos \frac{n\pi x_w}{x_e} \cos \frac{n\pi x}{x_e} \right\} \cdot \left\{ 1 + 2 \sum_{m=1}^{\infty} \exp \left[-\frac{m^2 \pi^2 \eta_y t}{y_e^2} \right] \cos \frac{m\pi y_w}{y_e} \cos \frac{m\pi y}{y_e} \right\} \cdot \left\{ 1 + 2 \sum_{l=1}^{\infty} \exp \left[-\frac{l^2 \pi^2 \eta_z t}{h^2} \right] \cos \frac{l\pi z_w}{h} \cos \frac{l\pi z}{h} \right\} \quad (12)$$

Substituting in the P_D expression, and simplifying equation (4) we obtain;

$$P_D = \frac{2\pi}{x_{eD} y_{eD}} \int_0^{t_D} \left\{ \left[1 + \frac{2x_{eD}}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \exp \left(-\frac{n^2 \pi^2 \tau_D}{x_{eD}^2} \right) \sin \frac{n\pi}{x_{eD}} \cos \frac{n\pi x_{wD}}{x_{eD}} \cos \frac{n\pi x_D}{x_{eD}} \right] \cdot \left[1 + 2 \sum_{m=1}^{\infty} \exp \left(-\frac{m^2 \pi^2 \tau_D}{y_{eD}^2} \right) \cos \frac{m\pi y_{wD}}{y_{eD}} \cos \frac{m\pi y_D}{y_{eD}} \right] \cdot \left[1 + 2 \sum_{l=1}^{\infty} \exp \left(-\frac{l^2 \pi^2 \tau_D}{h_D^2} \right) \cos \frac{l\pi z_{wD}}{h_D} \cos \frac{l\pi z_D}{h_D} \right] \right\} d\tau_D \quad (13)$$

To be able to analyse the slope and thus identify flow regimes, the dimensionless pressure derivative for equation (13) is given by;

$$P_D' = \frac{2\pi}{x_{eD}y_{eD}} \left\{ \left[1 + \frac{2x_{eD}}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \exp\left(-\frac{n^2\pi^2 t_D}{x_{eD}^2}\right) \sin \frac{n\pi}{x_{eD}} \cos \frac{n\pi x_{wD}}{x_{eD}} \cos \frac{n\pi x_D}{x_{eD}} \right] \cdot \left[1 + 2 \sum_{m=1}^{\infty} \exp\left(-\frac{m^2\pi^2 t_D}{y_{eD}^2}\right) \cos \frac{m\pi y_{wD}}{y_{eD}} \cos \frac{m\pi y_D}{y_{eD}} \right] \cdot \left[1 + 2 \sum_{l=1}^{\infty} \exp\left(-\frac{l^2\pi^2 t_D}{h_D^2}\right) \cos \frac{l\pi z_{wD}}{h_D} \cos \frac{l\pi z_D}{h_D} \right] \right\} t_D \quad (14)$$

The dimensionless variables used in this derivation are defined as follows;

$$i_D = \frac{2i}{L} \sqrt{\frac{k}{k_i}} \quad (15)$$

$$i_{wD} = \frac{2i_w}{L} \sqrt{\frac{k}{k_i}}, \quad (16)$$

$$i_{eD} = \frac{2i_e}{L} \sqrt{\frac{k}{k_i}}, \quad (17)$$

Where $i = x, y, z$ and $z_e = h$

$$t_D = \frac{4kt}{\Phi\mu c_t L^2} \quad (18)$$

And

$$\eta_i = \frac{k_i}{\Phi\mu c_t} \quad (19)$$

Where η_i is the diffusivity constant in the i -axial directions, x , y and z .

This is a mathematical model that can be applied to approximate the pressure response when we consider a horizontal well inside a completely bounded oil reservoir. This applies when all the boundaries of the reservoir start influencing the flow and thus attaining a pseudosteady state flow. The model derived shows an inverse proportionality between the pressure response and both the width and length of the well. Thus, for any given oil reservoir, if the permeability is high, then an increased pressure response will be attained from short and thinner reservoirs and for low permeability reservoirs the same will be achieved from wide and longer reservoirs.

CONCLUSION

From the above discussion, it's evident that the model developed can;

(a) Approximate pressure response during late times,

(b) Approximate the reservoir geometry.

This will require a careful substitution of the oil reservoir parameters. The length of the well as it compares to the length, width and thickness of the reservoir will determine how long this flow period lasts for meaningful analysis. Since the model considers late times, the duration in which a well is considered to be productive is a factor for the pressure response. Thus, this model can be applied to enhance the performance of a well at late times.

Nomenclature

B	Oil formation volume factor, rbbl/stb
c_t	Formation compressibility, 1/psi
h	Reservoir pay thickness, ft
h_D	Dimensionless reservoir pay thickness
i	Axial flow directions x , y and z

k	Reservoir permeability, md
k_x	Permeability, x-axis, md
k_y	Permeability y-axis, md
k_z	Permeability z-axis, md
l	Well length, ft
P_D	Dimensionless pressure
q	Flow rate, bbl/day
s	Source
t	Time, hrs
t_D	Dimensionless form of time
x	Length, x-axis, ft
x_D	Dimensionless form of length
x_e	Reservoir length, ft
x_{eD}	Dimensionless reservoir length
x_w	Source coordinate, x-axis, ft
x_{wD}	Dimensionless form of source coordinate, x-axis
y	Width, y-axis, ft
y_D	Dimensionless form of width
y_e	Reservoir width, ft
y_{eD}	Dimensionless reservoir width
y_w	Source coordinate, y-direction, ft
y_{wD}	Dimensionless form of source coordinate, y-axis
z	Thickness, z-axis, ft
z_D	Dimensionless form of thickness
z_w	Source coordinate, z-axis, ft
z_{wD}	Dimensionless form of source coordinate, z-axis
η_i	Diffusivity constant, i-axis, md-psi/cp
Φ	Porosity
μ	Reservoir fluid viscosity, cp
τ_D	Dimensionless form of dummy variable for time

Superscripts

'	Derivative
---	------------

Subscripts

D	Dimensionless
e	External
i	Axial direction
w	Wellbore

REFERENCES

- Adewole, E. S. (2009). The use of source and Green's functions to model pressure distribution in a bounded layered reservoir with lateral walls, Part II: General solutions. *Journal of Nigerian Association of Mathematical Physics*, 15, 197–204.
- Adewole, E. S., & Olafuyi, O. A. (2010). The use of source and Green's functions to derive dimensionless pressure and dimensionless pressure derivative distribution of a two-layered reservoir Part I: A-Sha ed Architecture. *Journal of Mathematics and Technology*, 92–101.
- Al Rbeawi, S. J. H., & Tiab, D. (2011). *Transient pressure analysis of horizontal wells in a multi-boundary system*. In *SPE production and operations symposium*. Society of Petroleum Engineers.
- Babu, D. K., & Odeh, A. S. (1989). Productivity of a horizontal well. *SPE Res Eng*, 4 (4): 417–421. SPE-18298-PA. <http://dx.doi.org/10.2118/18298-PA>.
- Rosa, A. J., & Carvalho, R. D. S. (1989). A mathematical model for pressure evaluation in an Infinite-conductivity horizontal well. *SPE Formation Evaluation*, 4(04), 559–566.
- Carslaw, H. S., & Jaeger, J. C. (1959). *Conduction of heat in solids*. Oxford Univ. Press, New York, 510.
- Clonts, M. D., & Ramey Jr, H. J. (1986). *Pressure transient analysis for wells with horizontal drainholes*. In *SPE California regional meeting*. Society of Petroleum Engineers.
- Daviau, F., Mouronval, G., Bourdarot, G., & Curutchet, P. (1988). Pressure analysis for horizontal wells. *SPE Formation Evaluation*, 3(04), 716–724.
- Gringarten, A. C., & Ramey Jr, H. J. (1973). The use of source and Green's functions in solving unsteady-flow problems in reservoirs. *Society of Petroleum Engineers Journal*, 13(05), 285–296.
- Lee, W. J. (1982). *Well Testing*. Society of Petroleum Engineers of AIME, New York.
- Odeh, A. S., & Babu, D. K. (1989). *Transient flow behavior of horizontal wells, pressure drawdown, and buildup analysis*. In *SPE California regional meeting*. Society of Petroleum Engineers.
- Ozkan, E., Raghavan, R., & Joshi, S. D. (1987). *Horizontal well pressure analysis*. In *SPE California regional meeting*. Society of Petroleum Engineers.
- Matthews, C. S., & Russell, D. G. (1967). *Pressure buildup and flow tests in wells* (Vol. 1). Henry L. Doherty Memorial Fund of AIME.